Problem Set 1

1. Derive the potential temperature conservation equation (Eq. 2.25[1])

$$\frac{d\theta}{dt} = S_\theta + \kappa_T \nabla^2 \theta,$$

where $S_\theta = S_T \left(\frac{p}{p_0}\right)^{-R_d/c_p}$ from the energy conservation equation (Eq. 2.22):

$$\rho_d c_p \frac{dT}{dt} = \frac{dp}{dt} + \rho_d c_p S_T + \rho_d c_p \kappa_T \nabla^2 T.$$

2. CO_2 is a well-mixed gas in the lower atmosphere, meaning that its long-term mean mixing ratio does not vary with height. The global mean CO_2 molar mixing ratio was 396.5 μ mol mol[1] in 2013. Estimate the vertical CO_2 mass density gradient in the atmospheric boundary layer of the standard atmosphere. (Hints: In the standard atmosphere, the pressure and the temperature are 1013.2 hPa and 15.0°C at the sea level and 898.7 hPa and 8.5°C at the altitude of 1000 m. Use a typical water vapor mixing ratio of 15 g kg^{-1} for both heights.)

3. The daily mean water vapor flux is 0.074 g m^{-2} s^{-}1. How much water, in mm of water depth, is lost via evaporation in one day?

4. The horizontal velocity divergence $\partial\overline{u}/\partial x + \partial\overline{v}/\partial y$ is 2×10^{-6} s^{-1} in a mid-latitude anticyclone. Estimate the associated mean vertical velocity at the height of 20 m above the surface.

5. The total kinetic energy per unit mass of air is given by

$E_T = \frac{1}{2}(u^2 + v^2 + w^2)$

Using the Reynolds averaging rules, show that the mean total kinetic energy, $\overline{E_T}$, is the sum of the mean flow kinetic energy, $\overline{E}$, and the turbulent kinetic energy, $\overline{e}$:

$\overline{E_T} = \overline{E} + \overline{e},$

where

$\overline{E_T} = \frac{1}{2}\overline{(u^2 + v^2 + w^2)}$

$\overline{E} = \frac{1}{2}(\overline{u}^2 + \overline{v}^2 + \overline{w}^2)$

$\overline{e} = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$

6. In some climate models, a grid cell can have multiple subgrid surface types. Each surface interacts independently with the overlaying atmosphere through forcing variables specified at the first model grid level. This level is usually at the so-called blending height where the atmosphere is well mixed horizontally (i. e., no sub-grid variations at this height). Assume that a grid cell consists of a smooth (momentum

[1] see Xuhui Lee textbook

roughness $z_0 = 0.001$ m) and a rough surface ($z_0 = 0.50$ m), air stability is neutral, the blending height is 50 m, and wind speed at the blending height is 5 m s^{-1}. Calculate the friction velocity for each of the two surfaces. What is the wind speed at the 2-m height above these surfaces?